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Energy of a Graph associated with Tamil Nadu District Map

Abdullah Firdowsi*
Department of Artificial Intelligence and Data Science Karpagam College of Engineering (Affiliated to Anna University) Coimbatore, Tamil Nadu
717821i102@kce.ac.in

Madhanegha
Department of Artificial Intelligence and Data Science Karpagam College of Engineering (Affiliated to Anna University) Coimbatore, Tamil Nadu
717821i134@kce.ac.in

Thangapandi
Department of Artificial Intelligence and Data Science Karpagam College of Engineering (Affiliated to Anna University) Coimbatore, Tamil Nadu
717821i155@kce.ac.in

Abstract — In recent years, the energy of graphs finds many applications in Data Science. In this contribution, the graph associated with Tamil Nadu district map has been taken in consideration. The adjacency matrix of the graph associated with districts is formed and it is 32*32 square matrix. The energy graph of this adjacency matrix is computed and the significance of the energy is discussed in detail. It would be useful in understanding the characteristics of paths between districts under consideration.

Keywords — Undirected Graph, Eigenvalues, Adjacency Matrix, Graph Energy.

I. INTRODUCTION

In recent years, the studies on energies of graphs find significant applications in Artificial Intelligence and Machine Learning. In 1970s, Gutman [5] introduced the concept of the energy of a graph by an equation. Gutman et al. [10] observed that the graph theory concept had arisen from molecular bonds only. The origin of graph energy is Hückel molecular orbital (HMO) theory which forms a path in finding π electrons energy approximately in HydroCarbon Molecules [6]. Balakrishnan[2] introduced the definition for comparison between graphs as "Two graphs with the same number of vertices are equienergetic if they have the same energy". The Eigenvalue of a graph G is the Eigenvalue of its adjacency matrix is the theorem explained by Samir K. Vaidya[11]. Vijayakumar et al. [9] proved that the Eigenvalues of a graph are the Eigenvalues of its adjacency matrix and explained the analytic expressions for the energy of two classes of regular graphs. A large number (over a hundred) variants of graph energy have been proposed, based on matrices other than the adjacency matrix[7]. Veena Mathad[14] and Sultan Senan Mahde[14] introduced the minimum hub energy $E_h(G)$ of a graph G and computed the minimum hub energies of some standard graphs and several well-known families of graphs. $E_h(G)$ of Upper and Lower Bounds was established. Venkata Anusha et al. [15] constructed the definition for the Matrix Energy of Graph G, which is defined as the summation of absolute values of all singular values of a graph G. A study on Euler Graph and it's applications observed from the research paper authored by Ashish Kumar[1]. Toida[13] explained the Properties of an Euler graph. Euler graph means an even number of edges connected to each of its vertices.With the help of all the above mentioned articles, we understand and calculate the energy of the graph G.

II. DEFINITIONS AND RESULTS

Eigenvalue : Eigen is a German word, which means characteristic. Eigenvalue is the characteristic scalar value of the square matrix associated with the set of linear equations, satisfies the equation $AX = \lambda 88$. For each value of λ, there exists a corresponding non-zero column vector *'X'* called Eigenvector.

Undirected graph: A graph where edges have no direction and can be traversed in both directions.

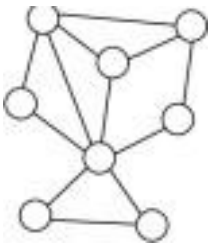

Fig. 1.1 Example for undirected graph

Finite graph: A countable number of edges and vertices is a finite graph. If the number of edges and the number of vertices that are present in a graph are finite in number, then that graph is called a finite graph.

Adjacency matrix: Each row and column of the matrix represents a vertex in the graph, and the value represents the edge between the two vertices. The adjacency matrix $A(G)$ of graph G is a square matrix of order n, whose (i, j) is equal to 1 if the vertices v_i and v_j are adjacent and otherwise it is equal to zero [1].

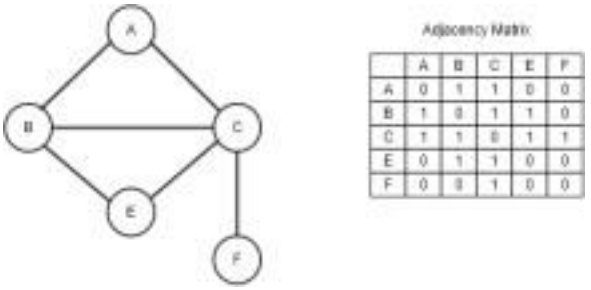

Fig. 1.2 Example for Adjacent Matrix of Undirected graph

Energy of a graph : Energy of a graph is a function that assigns a numerical value to a graph. It represents a measureof the stability or balance of the graph. The energy of a graph is used to describe certain properties of the graph or toquantify its structure and behaviour. The energy of a graph isthe sum of the absolute Eigenvalues of an adjacent matrix. The energy $E(G)$ is defined as

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

The Eigenvalues of an Adjacency Matrix A(G) are called an Eigenvalues of graph G. The set of the Eigenvaluesof a graph is the spectrum of the graph. The energy quantity is studied in the context of spectral graph theory.

III. FORMULATION AND DISCUSSION

Tamil Nadu is located in the southern part of India. Thereare 38 districts in TamilNadu. Capital of TamilNadu is Chennai. In 2019, the number of districts in TamilNadu increased from 32 to 38. We have taken only 32 districts into consideration since other 6 districts are emerging.

The corresponding finite undirected graph of TamilNadudistrict map as follows :

Fig. 1.3 Map of Tamil Nadu connecting Districts

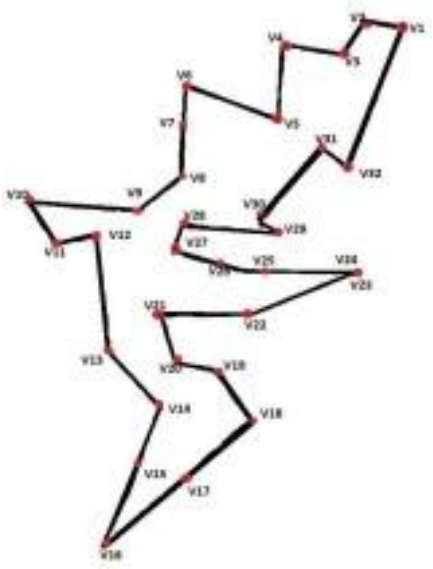

Fig. 1.4 Graph of Tamil Nadu District Map

The following is the list of districts considered for the current study :

v1 - Chennai v2 - Tiruvallur
v3 - Kancheepuramv4 - Vellore
v5 - Tiruvannamalaiv6 - Krishnagiri
v7 - Dharmapuri
v8 - Salemv9 - Erode
v10 - The Nilgirisv11 - Coimbatorev12 - Tiruppur v13 - Theni
v14 - Virudhunagarv15 - Tirunelveli v16 - Kanyakumariv17 - Thoothukudi
v18 - Ramanathapuramv19 - Sivagangai
v20 - Madurai v21 - Dindigul v22 - Pudukkottai
v23 - Nagapattinamv24 - Tiruvarur
v25 - Thanjavur
v26 - Tiruchirappalliv27 - Karur
v28 - Namakkal v29 - Ariyalur v30 - Perambalurv31 - Villupuramv32 - Cuddalore

The adjacency matrix of the graph is shown in Fig. 1.4 isas follows :

$$A(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 1 \\ 1 & 0 & 1 & 0 & \dots & 0 \\ 0 & 1 & 0 & 1 & \dots & 0 \\ \vdots & \cdots & \ddots & & 0 & \vdots \\ 0 & 0 & \dots & 0 & 1 & 0 \\ 0 & 0 & \dots & 1 & 0 & 1 \\ 1 & 0 & \dots & 0 & 1 & 0 \end{bmatrix}_{32 \times 32}$$

Clearly, it is a square matrix of order 32*32 and Eigenvaluesof the adjacent matrix are found, then absolute values of the eigenvalues are computed and the sum of the absolute valuesare calculated and that value is the energy of the graph of Tamil Nadu District Map,

CONCLUSION

In this current study, the graph associated with Tamil Nadu district map is considered. The adjacency matrix of the graph associated with districts is formed and its corresponding energy is calculated. The Energy of a graph of Tamil Nadu district shows that stability and structure is strong. All the properties possessed for an Eulerian Graph also hold good. It would be useful in understanding the characteristics of paths between districts under consideration.

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E(G) = 40.61628

The Energy of the graph of Tamil Nadu district shows that the stability and structure are strong. It satisfies the equation of deg(v) = 2. It is also noted that the degree of each vertex is all of even and it is an Eulerian Graph. Hence, All the properties possessed for an Eulerian Graph also hold that all of its vertices with nonzero degree belong to a single connected component. It is also observed that the graph of districts can be decomposed into edge disjoint cycles.

All the above computations are done with R programming toreduce the complexity.

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